



Solving the Poisson equation on irregularly shaped domains



Modern scientific simulations have enabled us to study non-linear phenomena that are impossible to study otherwise. Among the most challenging problems is the study of Einstein's theory of relativity which predicts the existence of gravitational waves detected by the LIGO collaboration. I am interested in recruiting a student interested in improving the elliptic solver of used to construct initial data for such simulations, to enable it to solve elliptic equations in irregularly shaped domains. Such domains occur when constructing initial data describing two neutron star, where elliptic equation have to be solved inside of the deformed stars. The project involves reviewing original mathematics, numerics and physics literature on methods related to the immersed surface method as well as developing a proof of concept code in python to demonstrate the performance of the methods. The successful applicant will be involved with the Relativity Group at NCSA and will be invited to participate in the weekly group meetings and discussions of their research projects.

Details: Constructing initial data for simulations of Einstein's theory of General Relativity typically involves solving an a elliptic equation of the generic form of:

$$\Delta \phi = -4 \pi \rho$$

in some domain Ω with a boundary condition on the edge of the domain $\partial\Omega$. Typically the coordinates used to discretize the problem are chosen to match with the domain, for example using spherical coordinates if Ω is a sphere. For very irregular Ω however it is convenient to stick to Cartesian coordinates x,y,z and instead *embed* the boundary $\partial\Omega$ into the solution domain by turning the condition in $\partial\Omega$ into a condition in Ω . Multiple methods to handle this were developed in engineering disciplines to handle irregular shaped domains and this project aims to investigate their suitability for numerical relativity using a toy equation and a 2-dimensional grid.

This project requires a solid understanding of partial differential equations and in ideally boundary value problems such as encounter in electrostatics. A good working knowledge of python in particular its numpy, scipy and matplotlib packages is essential to succeed in the project. The ability to read scientific literature and to develop code based on the information in the literature is key to this project.

Before applying to the project, please work through the exercise at: <https://wiki.ncsa.illinois.edu/display/~rhaas/SPIN+2019>