Asynchronous iterative algorithms & Reliability

Mourad Hakem

LIFC Laboratory - University of Franche Comté - France

AND Team

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Motivation

- Solve large linear / non-linear equation systems
- Result in problems composed of **millions of components**
- Single computing unit ? — Parallel and distributed platforms

Direct and iterative resolution methods

Equation systems can be solved with direct and iterative methods.

Direct method

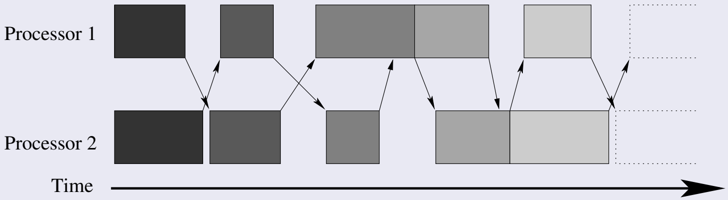
- Gives the **exact solution** for a problem after executing a finite number of operations.
- Does not solve all kinds of problems.
- Not well adapted for large problems.

Iterative method

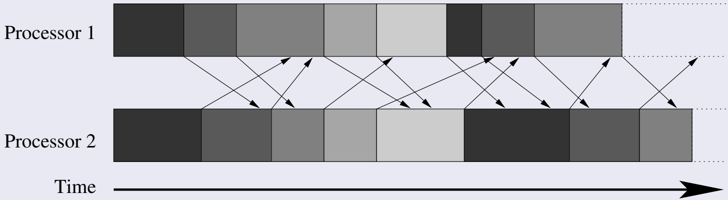
- Iterates until it gives a **good approximation** of the solution.
- Sometimes it is the only way to solve a problem.
- Well adapted for large problems and easy to prallelize.

Iterative methods : Synchronous vs Asynchronous iteration model

Synchronous iteration model



Asynchronous iteration model



Asynchronous iteration model

Advantages

- Eliminates idle times.
- Eliminates synchronizations.
- Tolerates message loss.
- Adapted for heterogeneous and volatile environments.

Difficulties

- Requires a specified asynchronous messaging mechanism.
- Hard to detect the global convergence.

All the difficulties can be overcome with a dedicated platforms:

JACE and JACEP2P-V2

Iterative model

To solve $Ax = b$,

Splitt $A = M - N$, M nonsingular

$$\text{Sequential case:} \quad \begin{cases} given\ x^0 \\ k = 1, 2, ... \\ x^{k+1} = M^{-1}Nx^k + M^{-1}b \\ \equiv x^{k+1} = F(x^k) \end{cases} \qquad (1)$$

Asynchronous iteration model

The asynchronous model is defined by

$$\left\{\begin{array}{l} \textit{Given } X^0 = (X_1^0, ..., X_m^0) \\ \quad \textit{for } k = 1, 2... \\ \qquad \textit{for } i = 1, ..., m \\ \qquad\quad X_i^{k+1} = \left\{\begin{array}{r} F_i(X_1^{r_1^i(k)}, ..., X_m^{r_m^i(k)}) \textit{ if } i \in S(k) \\ X_i^k \textit{ if } i \notin S(k) \end{array}\right. \end{array}\right. \tag{2}$$

- $S(k)$ is the set of components to be updated at step k
- $k - r_j^i(k)$ is the delay of the j^{th} proc.
  The i proc. computing the i^{th} block at the k^{th} iteration
- If $r_j^i(k) = k$ then: synchronous algorithms.

Convergence conditions

Theoritical results

- $\lim_{k\to\infty}\left(M^{-1}N\right)^k = 0 \quad \Leftrightarrow \rho\left(M^{-1}N\right) < 1$
- Synchronous iterations:
  $\rho\left(M^{-1}N\right) < 1 \Rightarrow (1)$ *converges to* $A^{-1}b$
- Asynchronous iterations:
  $\rho\left(\left|M^{-1}N\right|\right) < 1 \Rightarrow (2)$ *converges to* $A^{-1}b$

Motivation

Context

- Heterogeneous platforms - Clusters and Grids
- Iterative applications

Failures?

- Software is assumed to be reliable
- Only hardware failures of nodes
- Faults are assumed to be fail-silent/fail-stop

Fault tolerance objective?

tolerate at most r (reliability factor) node failures

Problem and solutions

Bi-criteria problem

latency & reliability : antagonist objectives

Solutions:

- checkpointing
- task replication
  - Primary/Backup (passive replication)
    - backup is activated only if the fault occurs
    - requires *fault detection/recovery mechanism*
  - Active replication
    - all replicas are activated in parallel
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Increasing reliability

- Without replication
  $\mathcal{R}_{\mathcal{A}} = \prod_{i=1}^{n} \mathcal{R}_i$
- With replication

$$\mathcal{R}_{\mathcal{A}} = \prod_{i} \mathcal{R}_i = \prod_{i} \left(1 - \prod_{j} (1 - \mathcal{R}_j)\right), 1 \leq i \leq n, 1 \leq j \leq r+1$$

A brief description of FT-Jace environment

FT-Jace: Reliable Parallel Programming Model for Distributed Computing Environments

Principle

- Multi-threaded
- Convergence detection - Asynchronous iteration model
- Uses the active software *replication scheme* to *mask failures*
- Can tolerate a given number r of arbitrary node failures

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Communication overhead reduction

Communication overhead: $e \rightarrow e(r+1)^2$

– Duplicating each task $r+1$ times is an absolute requirement

– But duplicating each communication $(r+1)^2$ times is not mandatory

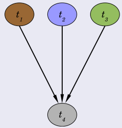
How to reduce communication overhead ?

Communication overhead reduction

All to ALL → One To One mapping ?

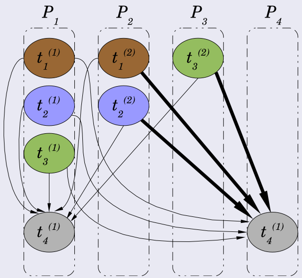
Idea: Try to decrease communication overhead from $e(r+1)^2$ down to at most $e(r+1)$

DAG



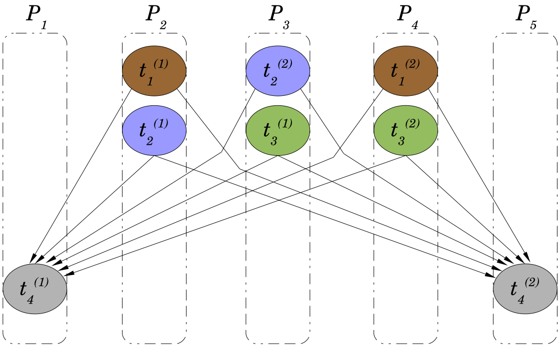
Property 1:

Let t and t_* two tasks involved in communication. If a replica of task t and a replica t_*^z of t_* are mapped on the same node P, then there is no need for other replicas of t_* to send data to node P.



Communication overhead reduction

Counter example



Communication overhead reduction

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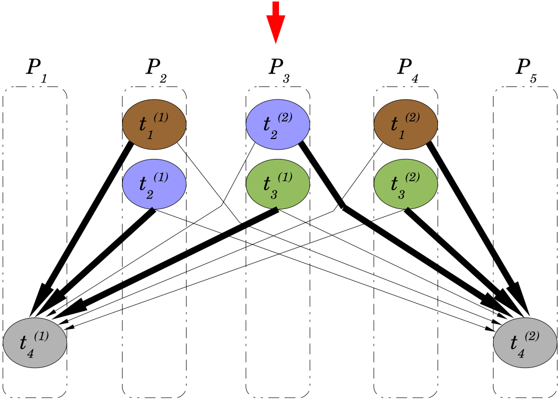

Communication overhead reduction

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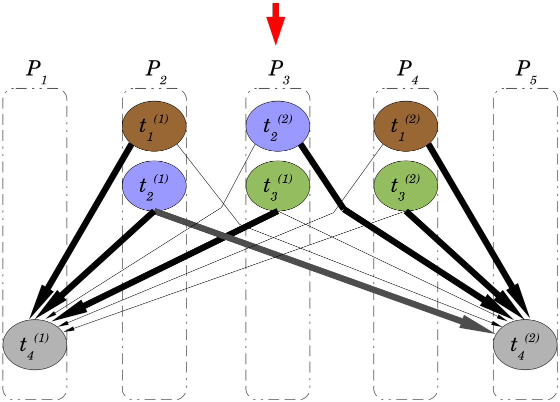
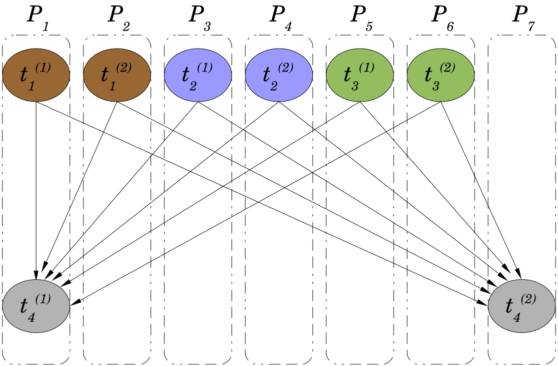


Communication overhead reduction

The number of communications will also be bounded by $e(r+1)$ if at each step replicas are assigned to different processors.



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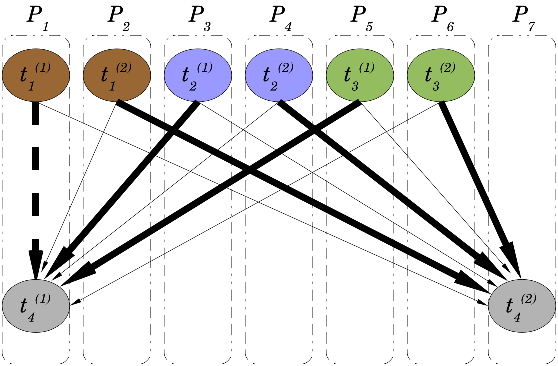


Some theoritical results

Theoritical results (Joint work with A. Benoit and Y. Robert)

- Fork/Outforest graph applications

$$e(r+1)$$

- Classical kernels/Parallel algorithms:
  LU, LAPLACE, STENCIL,DOOLITTLE,LDMt

$$V_2(r+1) + V_3\left(r\left\lceil\frac{(r+2)}{2}\right\rceil + 2\right),\ V_2 \leq \lfloor\frac{e}{2}\rfloor,\ V_3 \leq \lfloor\frac{e}{3}\rfloor$$

- General graph applications:

$$e\left(r\left\lceil\frac{(r+2)}{2}\right\rceil + 1\right)$$

Experimental results

Aim

- Evaluation of FT-Jace performance
- Comparison with fault-free version ($r = 0$)

Parameters

- Frensh Grid'5000 platform
- 50 nodes, bi-core Opteron 2.0 GHZ – Gigabit Ethernet
- Jacobi method to solve linear systems ($Ax = b$)
  (**Synchronous & Asynchronous mode**)
- size of the matrix A is $10^8 \times 10^8$.
- $r = \{1, 2, \ldots, 10\}$

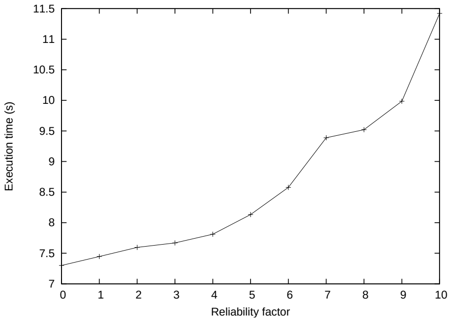
Metrics

- Latency
- Overhead

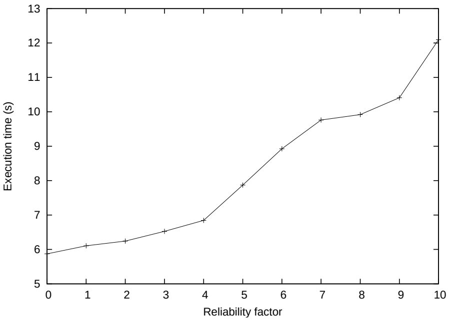
Experimental results

The achieved latency

Asynchronous mode



Synchronous mode



Experimental results

Table: Fault tolerance overhead (%)

Reliability factor (r)	2	4	6	8	10
Asynchronous mode	0.04	0.06	0.17	0.30	0.56
Synchronous mode	0.06	0.16	0.52	0.69	1.06

Fault tolerance overhead increases slightly
as reliability factor goes up

Conclusion

Conclusion

- Iterative computing model
- Asynchronous model is reliable by nature
- Reliability: **FT-Jace** environment
- Good performance

Perspective

- Effective scheduling strategy
- Checkpointing ?