Numerical Optimization for Automatic Tuning of Codes

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Motivation: A Looming Storm for Software & Libraries

Architectures are getting increasingly complex

- Multiple cores, deep memory hierarchies, software-controlled storage, shared resources, SIMD compute engines, heterogeneity, ...

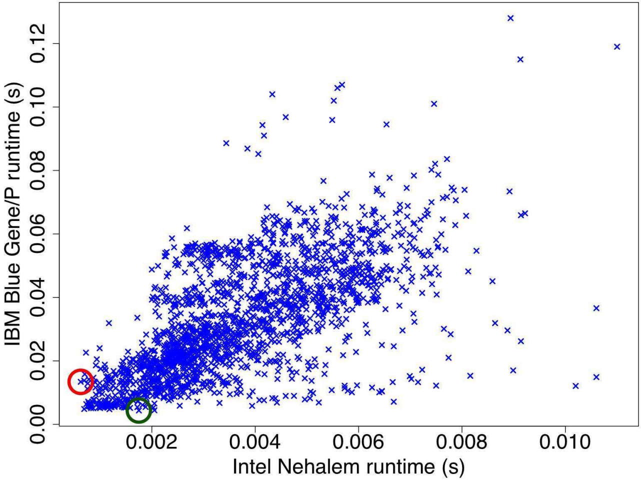
Performance optimization is getting more important

- Today's sequential and parallel applications may not be faster on tomorrow's architectures
- Growing complexity of scientific applications: tradeoffs between performance and maintainability (e.g., good software engineering practices)
- Managing data locality at least as important as optimizing parallelism
- Managing power of growing importance

Performance portability

- Tuning for a particular architecture potentially hinders performance on other architectures

Overtuning Can Destroy Performance Portability



Each × denotes a DGEMM variant

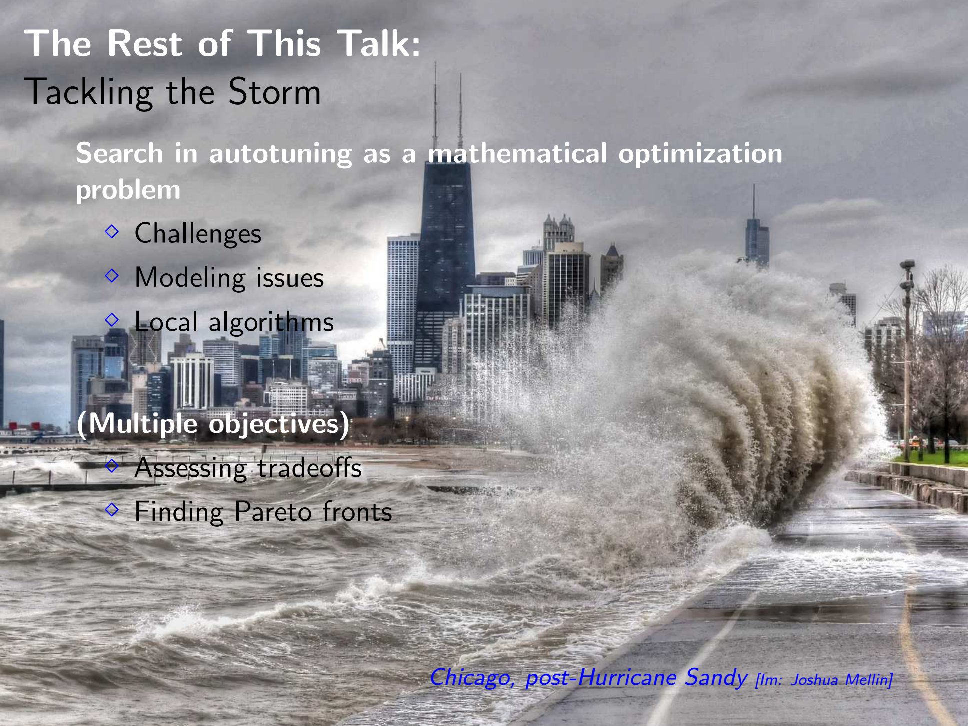
The Rest of This Talk:
Tackling the Storm

Search in autotuning as a mathematical optimization problem

- Challenges
- Modeling issues
- Local algorithms

(Multiple objectives)

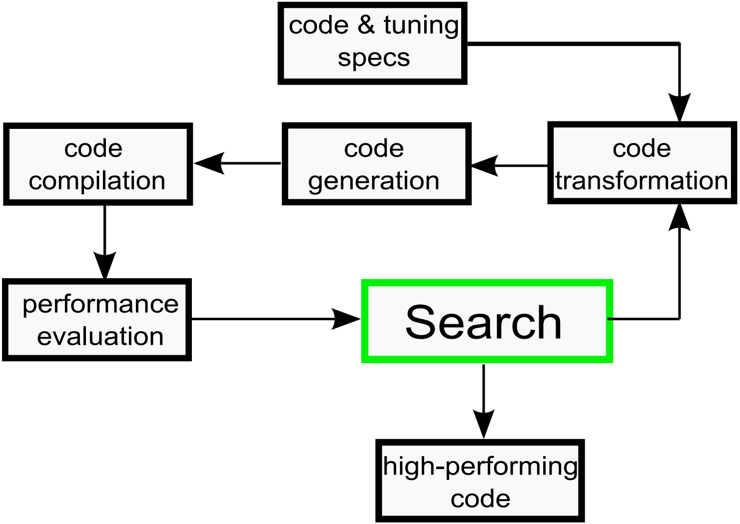
- Assessing tradeoffs
- Finding Pareto fronts



Chicago, post-Hurricane Sandy [Im: Joshua Mellin]

Automating Empirical Performance Tuning

Given a computation kernel and transformation space:



Search in Autotuning

Alternatives:

- Complete enumeration
  - Prohibitively expensive (10^{50} variants!)
  - Unnecessary?
- Pruning
  - Careful balancing act (between aggressive and conservative strategies)

Helpful (necessary?) precursors:

The expert still plays a role!

- Identify variable space (parameters to be tuned, ranges, constraints)
- Quantify measurement limitations and noise
- Incorporate known theoretical considerations (models)
- Construct meaningful objectives

→ Reduce search space and/or number of variants that need to be examined

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Our goal

Design, implement, and analyze *efficient optimization (=search) algorithms*

. . . for tuning kernels in small computation budgets

Is a Sophisticated Search Algorithm Needed?

[Seymour, You, & Dongarra, Cluster Computing '08]: Random search performs better than alternatives as the number of tuning parameters grows **!**

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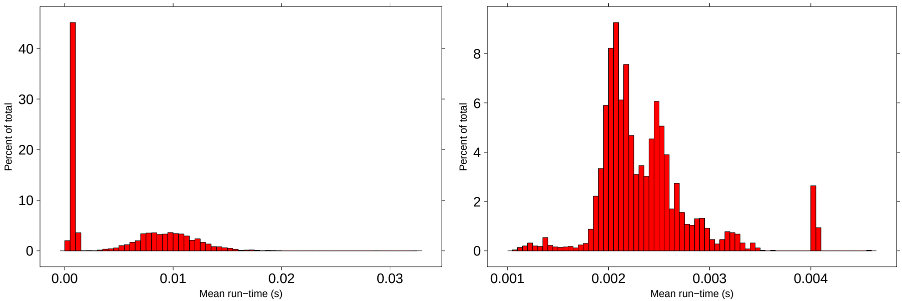
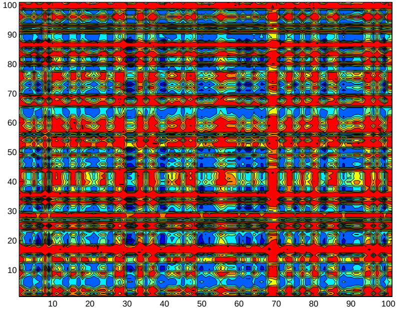
Depends on distribution of high-performing variants:

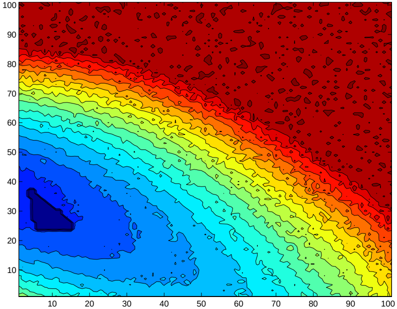


(5000 semantically equivalent variants each)

Is a Sophisticated Search Algorithm Useful?

Depends on structure of the (modeled) search space:





Both 2-dimensional problems have the same histogram

Must learn/model/exploit this structure to quickly find high-performing variants

Formulation and Modeling: Optimization is Optimization

Finding the best configuration is a mathematical optimization problem

$$\min_x \{f(x) : x = (x_\mathcal{I}, x_\mathcal{B}, x_\mathcal{C}) \in \mathcal{D} \subset \mathbb{R}^n\}$$

x multidimensional parameterization (compiler type, compiler flags, unroll/tiling factors, internal tolerances, ...) for a code variant

$f(x)$ empirical performance metric of x such as FLOPS, power, or run time (requires a run)

$\mathcal{D}$ search domain (constraints for feasible transformation, no errors, ...)

- bound: unroll $\in [1, \ldots, 30]$; RT $= 2^i$, i=[0,1,2,3]
- known: $(RT_I * RT_J \leq 150)$ (cheap); power consumption ≤ 90 W (expensive)
- hidden: transformation errors (relatively cheap), compilation (expensive), and run time (very expensive) failures

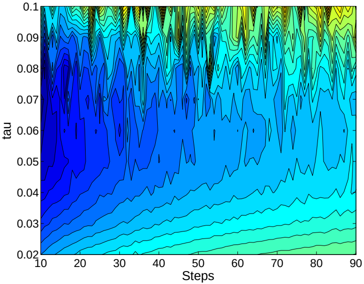
See [Balaprakash, Hovland, & W., iWAPT '11]

Optimization Challenges in Autotuning

$\min_x \{f(x) : x = (x_{\mathcal{I}}, x_{\mathcal{B}}, x_{\mathcal{C}}) \in \mathcal{D} \subset \mathbb{R}^n\}$

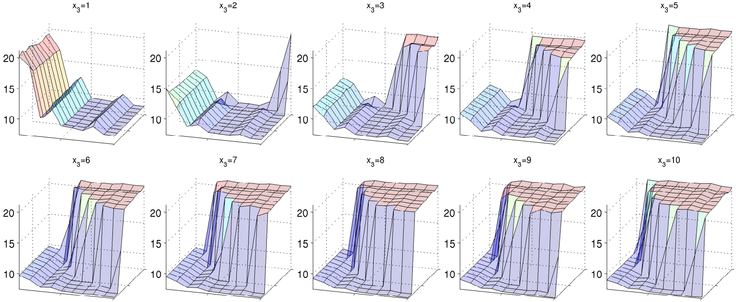
- f noisy, expensive, black box
- Discrete x unrelaxable
- $\nabla_x f$ unavailable/nonexistent
- "Cliffs", many distinct/local solutions?

Calls for **Derivative-Free Optimization**



↓ Integer Space: `MM` (MatMult)

↑ Mixed-Integer: Lattice QCD code



Search Problems in Automatic Performance Tuning

Problems: SPAPT [Balaprakash, Norris, & W., ICCS '12]

- 12 kernel codes
  - elementary linear algebra, linear solver, stencil codes, and elementary statistical computing configurations
- SPAPT problem = code + set of transformations + parameter specifications + constraints + input size

Code transformation: Orio performance tuning framework

Performance metric $f(x)$: mean run time of 10 runs

Kernel	Transformations	n_i	n_b	$\|\mathcal{D}\|$
ADI	CT, RT, UJ	16	4	2.818e+21
ATAX	CT, RT, UJ	13	6	6.115e+17
BiCG	CT, RT, UJ	9	4	2.654e+12
COR	CT, RT, UJ	16	4	2.818e+21
DGEMV	CT, RT, UJ	38	11	1.241e+53
FDTD4d2d	CT, RT, UJ	25	5	1.616e+33
GEMVER	CT, RT, UJ	17	6	1.409e+23
GESUMMV	CT, RT, UJ	8	3	5.308e+10
HMC	CT, RT, UJ	7	8	5.308e+10
Jacobi-1d	CT, RT, UJ	8	3	5.308e+10
LU	CT, RT, UJ	9	5	2.654e+12
MM	CT, RT, UJ	10	4	3.732e+11

SPAPT: Orio-ready Implementation

[Norris, Hartono, & Gropp, '07]

- Extensible empirical tuning system
- Allows inserting annotations as structured comments
- Supports architecture independent and specific optimizations

```
/* AXPY Kernel */
for (i=0; i<=n-1; i++)
   y[i]=y[i]+a1*x1[i]+a2*x2[i]+a3*x3[i]+a4*x4[i];
```

↓

/* Tuning specifications */ UF = {1,. . . ,30}; PAR = {True, False}

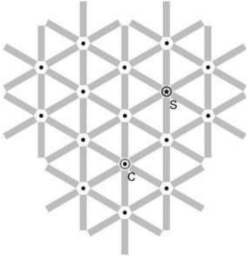
```
/*@ begin Loop (
  transform Unroll(ufactor=UF, parallelize=PAR)
    for (i=0; i<=n-1; i++)
       y[i]=y[i]+a1*x1[i]+a2*x2[i]+a3*x3[i]+a4*x4[i];
      )
@*/
```

Derivative-Free Optimization Algorithms

"Dibs:" An algorithm for Chicago winter parking reservations

Classical Algorithms for Performance Tuning

Global search



◇ exploration and exploitation
◇ find the globally best*
◇ long search time
◇ parameter sensitive

Local search

◇ limited exploration
◇ find the locally best
◇ short search time
◇ risk of bad local solution

Hypothesis: customized local search algorithms are effective for short computational budgets

Previous Algorithms for Performance Tuning

[Seymour, You, & Dongarra, Cluster Computing '08] and [Kisuki, Knijnenburg, & O'Boyle, PACT '00] compared several global and local algorithms

- Random search outperforms a genetic algorithm, simulated annealing, particle swarm, Nelder-Mead, and orthogonal search !
- Large number of high-performing parameter configurations → easy to find one of them

[Norris, Hartono, & Gropp, *Computational Science* '07] used several global and local algorithms but no comparison

- Nelder-Mead simplex method, simulated annealing, a genetic algorithm

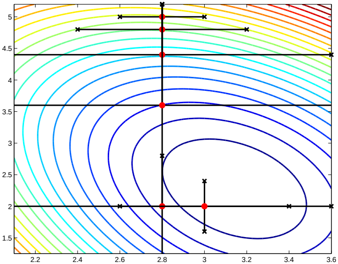
Other local search algorithms without comparison to global search:

- Orthogonal search in ATLAS [Whaley & Dongarra, SC '98]
- Pattern search in loop optimization [Qasem, Kennedy, & Mellor-Crummey SC '06]
- Modified Nelder-Mead simplex algorithm in Active Harmony [Tiwari, Chen, Chame, Hall, & Hollingsworth, IPDPS '09]

Local Algorithms: Direct Search Methods

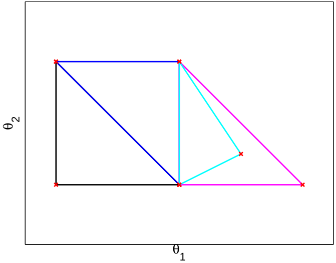
See [Kolda, Lewis, & Torczon, *SIREV* '03]

Pattern Search



Easy to parallelize f evaluations

Nelder-Mead

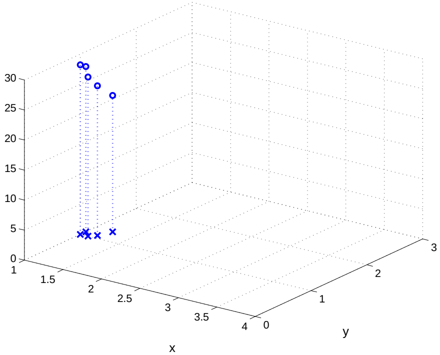


Popularized by *Numerical Recipes*

- Rely on indicator functions: $[f(x_k + s) <^? f(x_k)]$
  - Ignore valuable information on relative magnitudes of $f(x_k)$

Making the Most of Little Information on f

- f is expensive $\Rightarrow$ can afford to make better use of points
- Overhead of the optimization routine is minimal (negligible?) relative to cost of empirical evaluation



Bank of data, $\{x_i, f(x_i)\}_{i=1}^{k}$:

$=$ Everything* known about f

Idea:

- Make use of growing bank as optimization progresses

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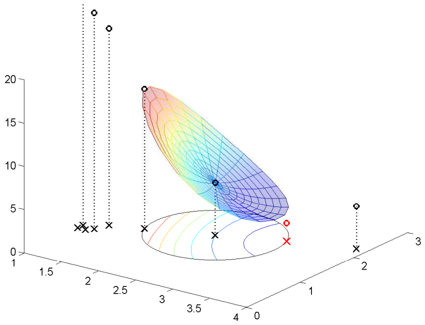

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= Everything* known about f

Idea:

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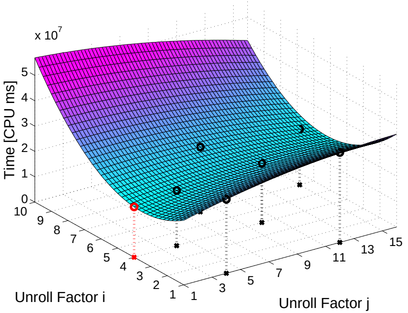
Surrogate-Based Trust-Region Algorithms

Substitute $\min \{m(x) : x \in \mathcal{B}_k\}$ for $\min f(x)$

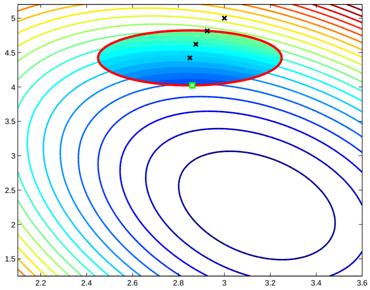
f expensive, no ∇f

m cheap, analytic derivatives

Surrogate based on known f values

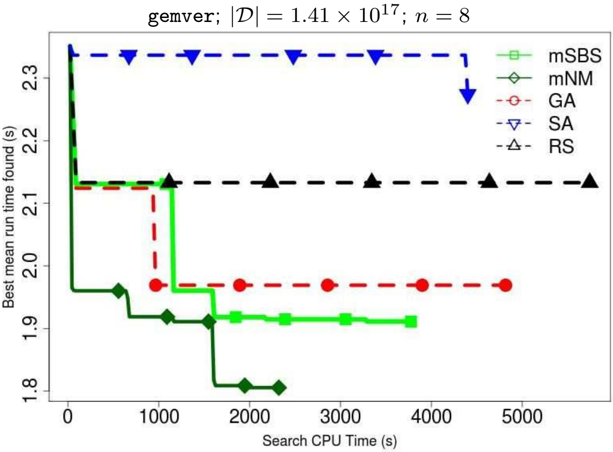


Trust $m \approx f$ in $\mathcal{B}_k$



Surrogates: predict improvement

Sample Comparison: Unoptimized Default Starting Point

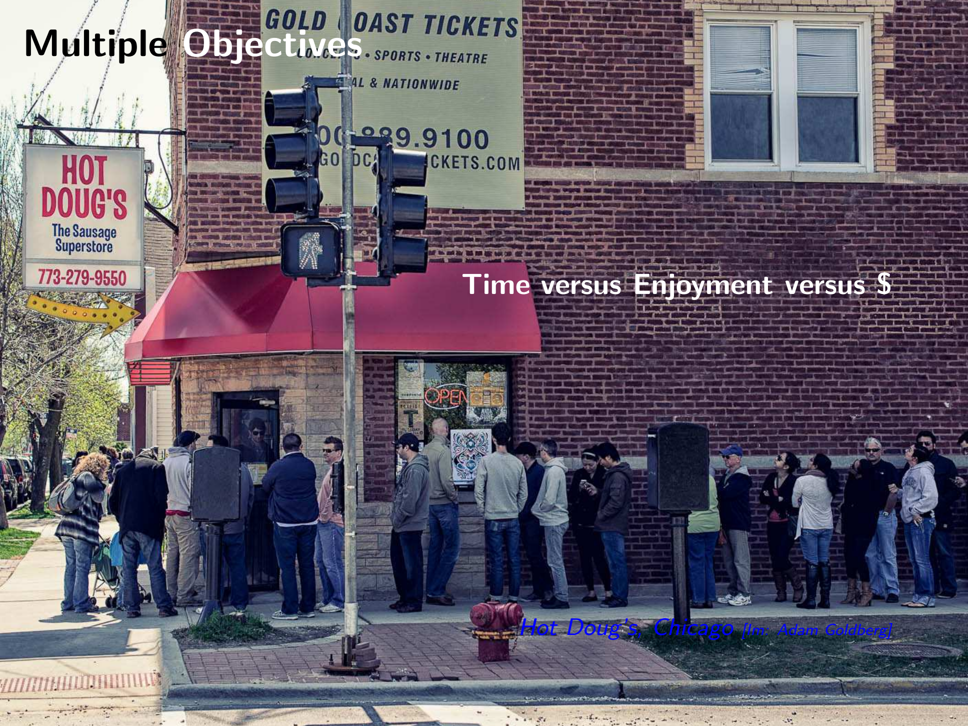


- **Double win: Better solutions, less time** (=time/evaluation)
- 10/12 SPAPT problems local search outperforms global search

(Down and to the left is better; Markers every 20 evaluations)

See [Balaprakash, Hovland, & W., VecPar '12]

Multiple Objectives



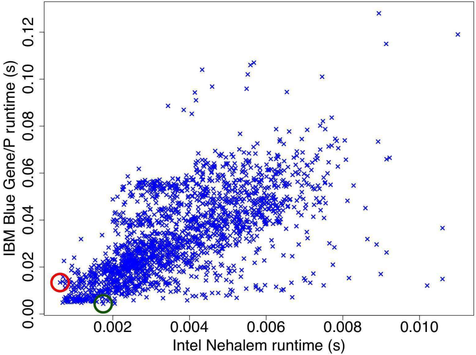
Time versus Enjoyment versus $

Hot Doug's, Chicago [Im: Adam Goldberg]

Simultaneously Optimizing Multiple Objectives

$$\min_{x\in\mathcal{D}}\{f_1(x), f_2(x), \ldots, f_p(x)\}$$

- No *a priori* weights w_i $\left(\sum_i w_i f_i(x)\right)$
- Dominated points $\tilde{x}$: $\exists x^* \in \mathcal{D}$ with
  $f_i(\tilde{x}) \geq f_i(x^*)\ \forall i,$
  $f_j(\tilde{x}) > f_j(x^*)$ some j
- Seek Pareto front of non-dominated points



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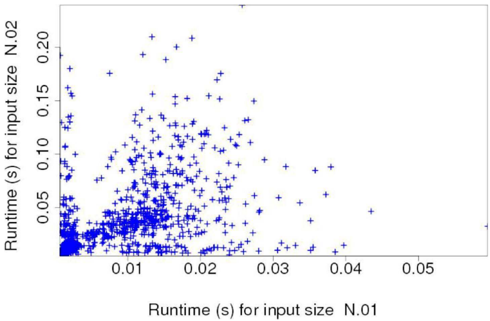
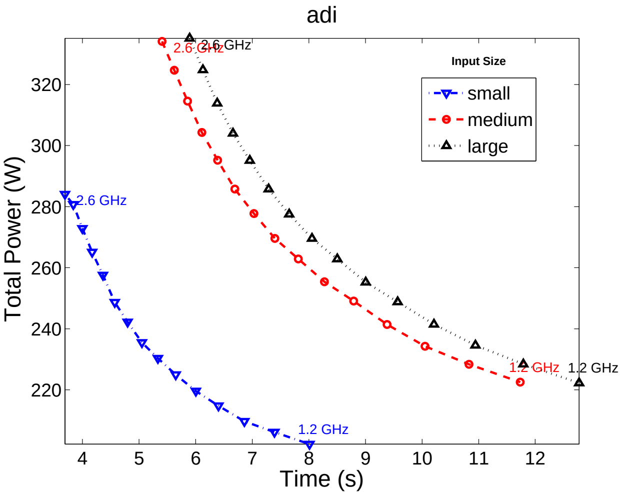


Multiple Objectives: Time, Power, Energy



◇ Tradeoffs in power do not imply tradeoffs in energy

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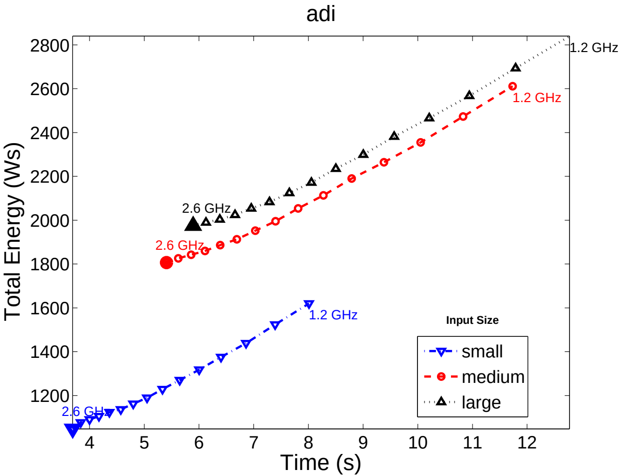

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- Objectives may not be conflicting: "Race to idle

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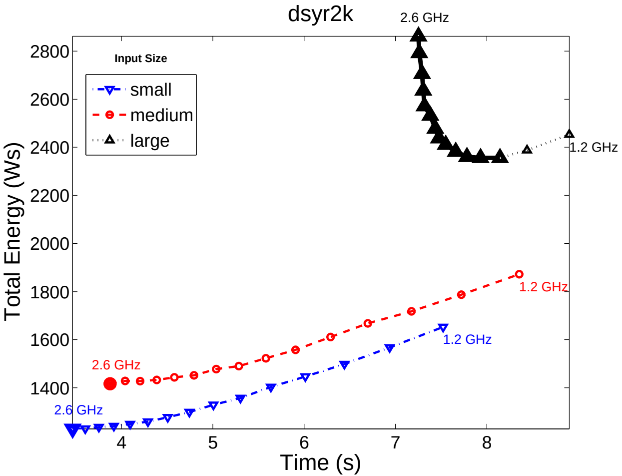

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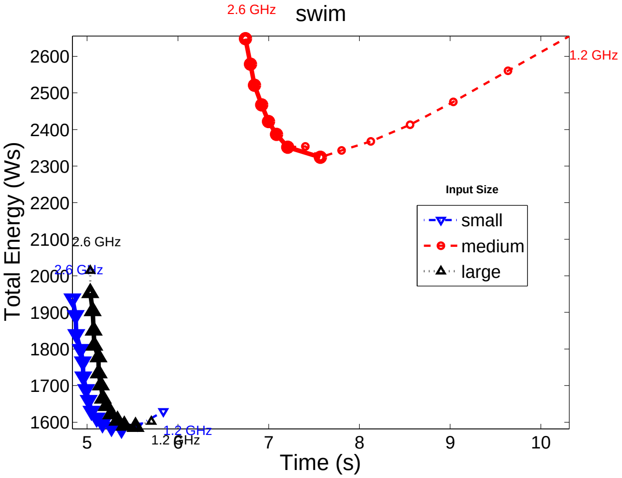

- Tradeoffs in power do not imply tradeoffs in energy
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- Tradeoffs occur at different frequencies

Summary and Links

- Performance tuning increasingly necessary, not yet "automatic"
- Derivative-free optimization is a powerful, practical tool

When the available tuning time is limited:

- Global exploration less useful
- Problem formulation and starting point play important roles

Future work includes:

- Incorporation of models, binary parameters, constraints (from models or otherwise), online restart strategies,role in full application codes, . . .

→ always collecting new search/optimization problems

. . . especially those with structure

Some preprints `http://mcs.anl.gov/~wild`

Orio `http://trac.mcs.anl.gov/projects/performance/wiki/Orio`

SPAPT `http://trac ··· /performance/browser/orio/testsuite/SPAPT.v.01`

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