

# Unified Model for Assessing Checkpointing Protocols at Extreme-Scale

George BOSILCA<sup>1</sup>, Aurélien BOUTEILLER<sup>1</sup>,  
Elisabeth BRUNET<sup>2</sup>, Franck CAPPELLO<sup>3</sup>,  
Jack DONGARRA<sup>1</sup>, [Amina GUERMOUCHE](#)<sup>4</sup>,  
Thomas HÉRAULT<sup>1</sup>, Yves ROBERT<sup>1,4</sup>,  
Frédéric VIVIEN<sup>4</sup>, and Dounia ZAIDOUNI<sup>4</sup>

1. University of Tennessee Knoxville, USA
2. Telecom SudParis, France
3. INRIA & University of Illinois at Urbana Champaign, USA
4. Ecole Normale Supérieure de Lyon & INRIA, France

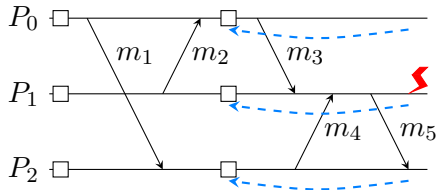
November 21, 2012

# Motivation

- **Very very** large number of processing elements (e.g.,  $2^{20}$ )  
⇒ Probability of failures dramatically increases
- Large application to be executed on whole platform  
⇒ Failure(s) will most likely occur before completion!
- Resilience provided through checkpointing
  - ① Coordinated protocols
  - ② Hierarchical protocols

# Coordinated Checkpointing Protocols

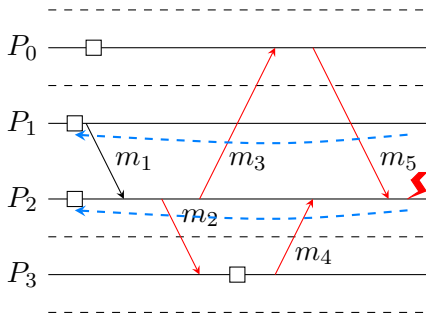
- Coordinated checkpoints over all processes
- Global restart after a failure



- 😊 No risk of cascading rollbacks
- 😊 No need to log messages
- 😞 All processors need to roll back

# Hierarchical Protocols

- Clusters of processes
- Coordinated checkpointing protocol within clusters
- Message logging protocols between clusters
- Only processors from failed group need to roll back



- ☹ Need to log inter-groups messages
  - Slows down failure-free execution
  - Increases checkpoint size/time
- 😊 Faster re-execution with logged messages

# Which checkpointing protocol to use?

## Coordinated checkpointing

- 😊 No risk of cascading rollbacks
- 😊 No need to log messages
- 😞 All processors need to roll back
- 😞 Rumor: May not scale to very large platforms

## Hierarchical checkpointing

- 😞 Need to log inter-groups messages
  - Slowdowns failure-free execution
  - Increases checkpoint size/time
- 😊 Only processors from failed group need to roll back
- 😊 Faster re-execution with logged messages
- 😊 Rumor: Should scale to very large platforms

# Outline

## ① Protocol Overhead

- Coordinated checkpointing

- Hierarchical checkpointing

## ② Accounting for message logging

## ③ Instantiating the model

- Applications

- Platforms

## ④ Experimental results

- Simulations

# Outline

- ① Protocol Overhead
- ② Accounting for message logging
- ③ Instantiating the model
- ④ Experimental results

# Framework

- Periodic checkpointing policies (of period  $T$ )
- Independent and identically distributed failures
- Platform failure inter-arrival time:  $\mu$
- Tightly-coupled application:  
    progress  $\Leftrightarrow$  all processors available
- First-order approximation: at most one failure within a period

**Waste:** fraction of time not spent for useful computations



# Waste

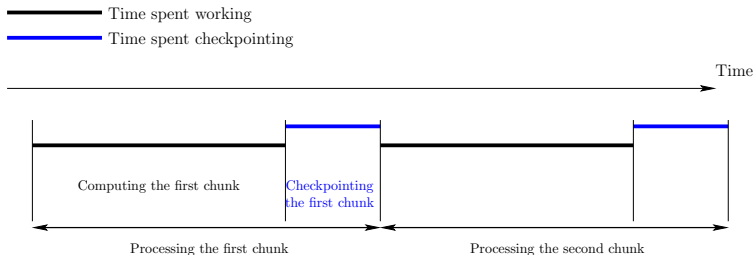
- $\text{TIME}_{\text{base}}$ : application base time
- $\text{TIME}_{\text{FF}}$ : with periodic checkpoints but failure-free
- $\text{TIME}_{\text{final}}$ : expectation of time with failures

$$(1 - \text{WASTE}[FF])\text{TIME}_{\text{FF}} = \text{TIME}_{\text{base}}$$

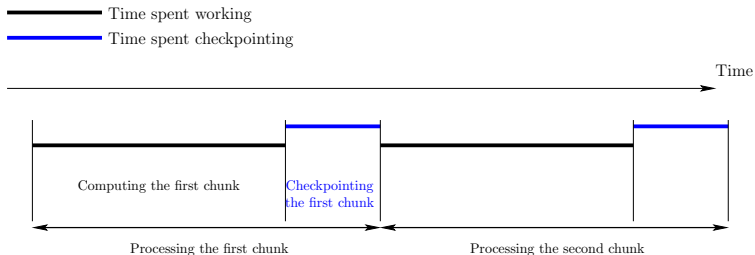
$$(1 - \text{WASTE}[fail])\text{TIME}_{\text{final}} = \text{TIME}_{\text{FF}}$$

$$1 - \text{WASTE} = 1 - (1 - \text{WASTE}[FF])(1 - \text{WASTE}[fail])$$

# Checkpointing cost

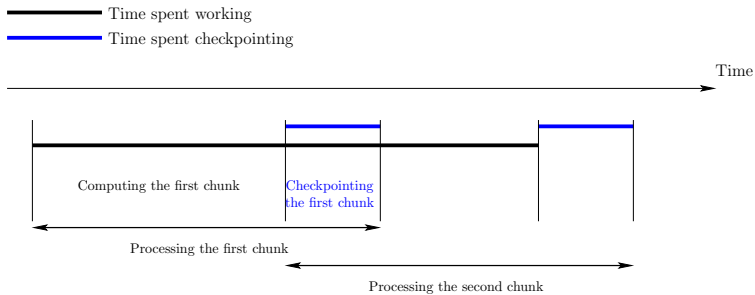


# Checkpointing cost



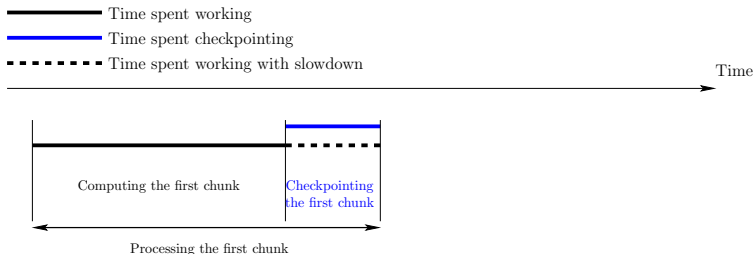
**Blocking model:** while a checkpoint is taken, no computation can be performed

# Checkpointing cost



**Non-blocking model:** while a checkpoint is taken, computations are not impacted (e.g., first copy state to RAM, then copy RAM to disk)

# Checkpointing cost



**General model:** while a checkpoint is taken, computations are slowed-down: during a checkpoint of duration  $C$ , the same amount of computation is done as during a time  $\alpha C$  without checkpointing ( $0 \leq \alpha \leq 1$ ).

## ① Protocol Overhead

Coordinated checkpointing

Hierarchical checkpointing

## ② Accounting for message logging

## ③ Instantiating the model

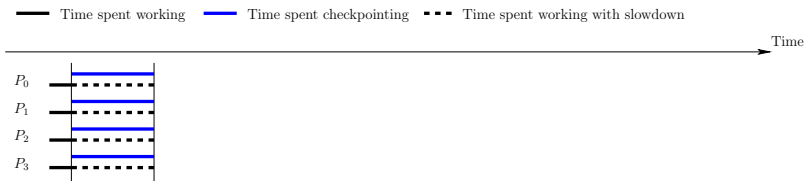
Applications

Platforms

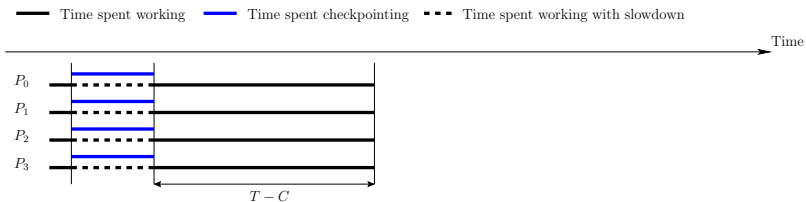
## ④ Experimental results

Simulations

# Waste in absence of failures

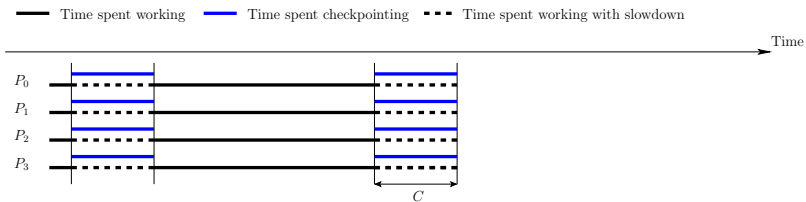


# Waste in absence of failures

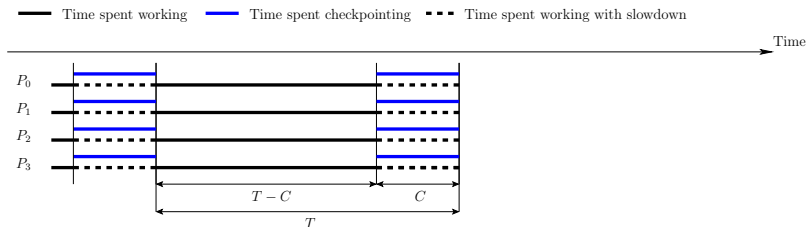




# Waste in absence of failures



# Waste in absence of failures

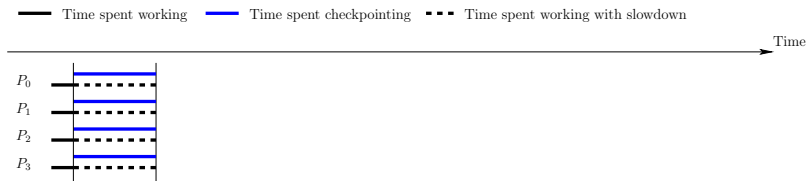


Time elapsed since last checkpoint:  $T$

Amount of computation saved:  $(T - C) + \alpha C$

$$\text{WASTE}[FF] = \frac{T - ((T - C) + \alpha C)}{T} = \frac{(1 - \alpha)C}{T}$$

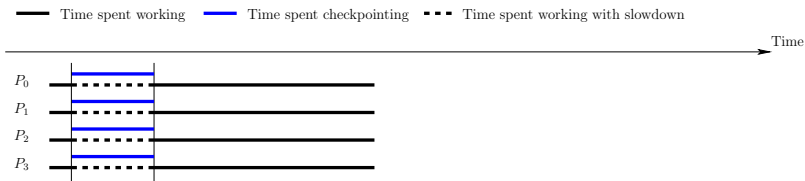
# Waste due to failures



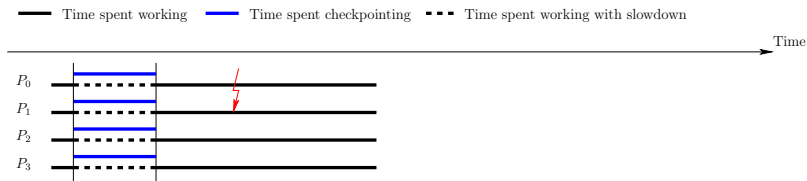
Failure can happen

- ① During computation phase
  - ② During checkpointing phase
- RE-EXEC: Time needed for the re-execution

# Waste due to failures in computation phase

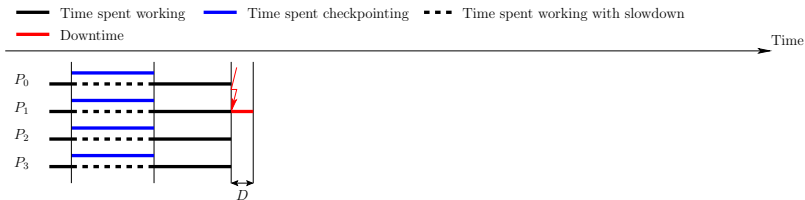


# Waste due to failures in computation phase

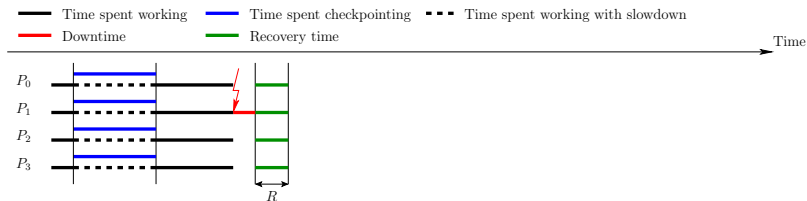


Coordinated checkpointing protocol: when one processor is victim of a failure, all processors lose their work and must roll-back to last checkpoint

# Waste due to failures in computation phase

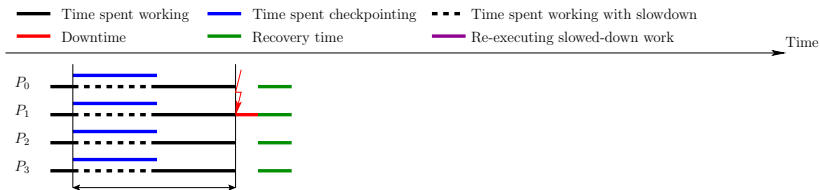


# Waste due to failures in computation phase



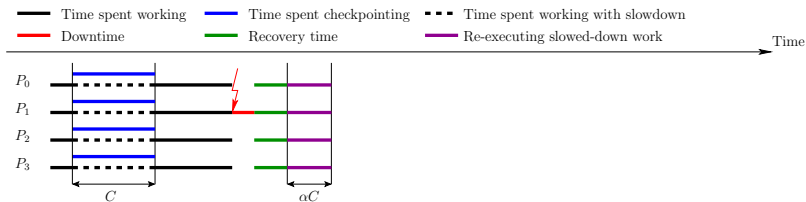
Coordinated checkpointing protocol: All processors must recover from last checkpoint

# Waste due to failures in computation phase





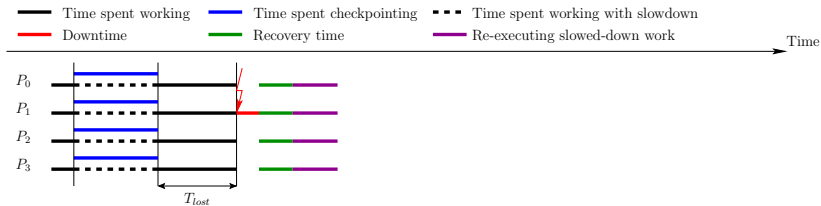
# Waste due to failures in computation phase



Redo the work destroyed by the failure, that was done in the checkpointing phase before the computation phase

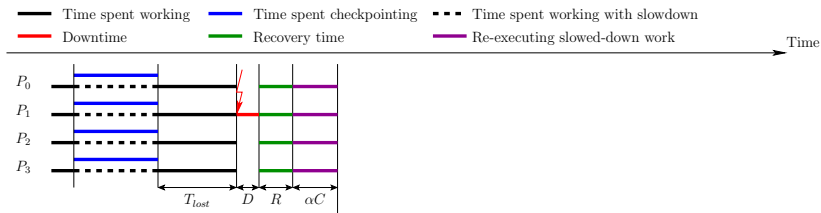
But no checkpoint is taken in parallel, hence this re-computation is faster than the original computation

# Waste due to failures in computation phase



Re-execute the computation phase

# Waste due to failures in computation phase

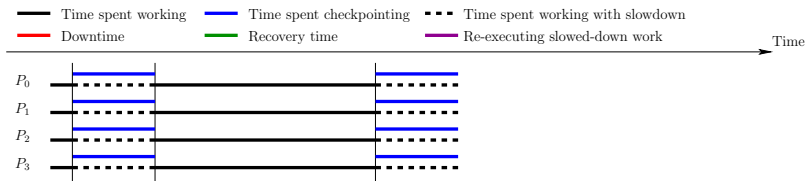


$$\text{RE-EXEC: } \text{RE-EXEC}_{\text{coord-fail-in-work}} = T_{lost} + \alpha C$$

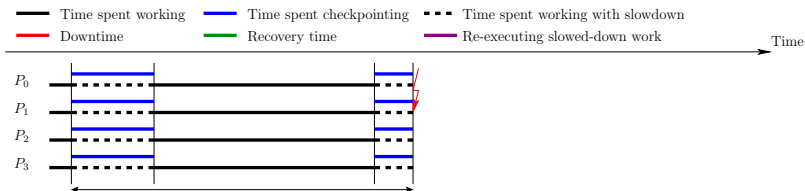
$$\text{Expectation: } T_{lost} = \frac{1}{2}(T - C)$$

$$\text{RE-EXEC}_{\text{coord-fail-in-work}} = \frac{T - C}{2} + \alpha C$$

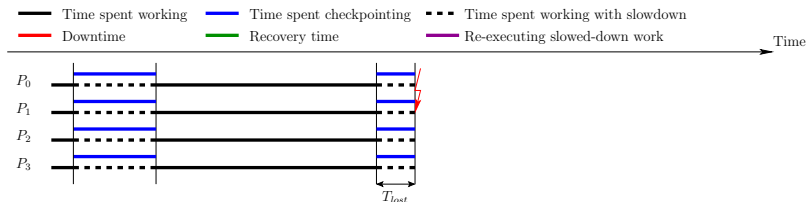
# Waste due to failures in checkpointing phase



# Waste due to failures in checkpointing phase



# Waste due to failures in checkpointing phase



$$\text{RE-EXEC}_{coord-fail-in-checkpoint} = (T - C) + T_{lost} + \alpha C$$

$$\text{Expectation: } T_{lost} = \frac{1}{2}C$$

$$\begin{aligned} \text{RE-EXEC}_{coord-fail-in-checkpoint} &= (T - C) + \frac{C}{2} + \alpha C \\ &= T - \frac{C}{2} + \alpha C \end{aligned}$$

# Waste due to failures

- Failure in the computation phase (probability:  $\frac{T-C}{T}$ )

$$\text{RE-EXEC}_{\text{coord-fail-in-work}} = \frac{T-C}{2} + \alpha C$$

- Failure in the checkpointing phase (probability:  $\frac{C}{T}$ )

$$\text{RE-EXEC}_{\text{coord-fail-in-checkpoint}} = T - \frac{C}{2} + \alpha C$$

$$\begin{aligned} \frac{T-C}{T} \left( \frac{T-C}{2} + \alpha C \right) + \frac{C}{T} \left( T - \frac{C}{2} + \alpha C \right) \\ = \alpha C + \frac{T}{2} \end{aligned}$$

# Total waste

$$\text{WASTE}[FF] = \frac{(1 - \alpha)C}{T}$$

$$\text{WASTE}[fail] = \frac{1}{\mu} \left( D + R + \alpha C + \frac{T}{2} \right)$$

$$\text{WASTE} = \text{WASTE}[FF] + \text{WASTE}[fail] - \text{WASTE}[FF]\text{WASTE}[fail]$$



# Total waste

$$\text{WASTE}[FF] = \frac{(1 - \alpha)C}{T}$$

$$\text{WASTE}[fail] = \frac{1}{\mu} \left( D + R + \alpha C + \frac{T}{2} \right)$$

$$\text{WASTE} = \text{WASTE}[FF] + \text{WASTE}[fail] - \text{WASTE}[FF]\text{WASTE}[fail]$$

## Optimal period

$$T^* = \sqrt{2(1 - \alpha)(\mu - (D + R))C}$$

## ① Protocol Overhead

Coordinated checkpointing

Hierarchical checkpointing

## ② Accounting for message logging

## ③ Instantiating the model

Applications

Platforms

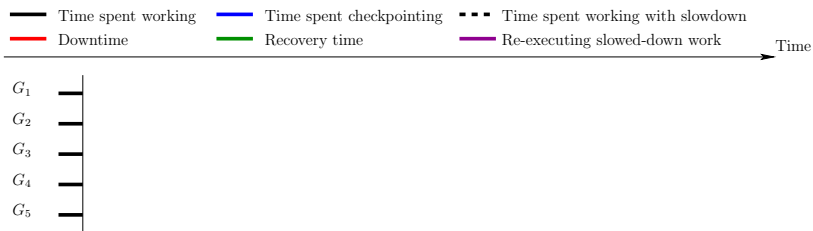
## ④ Experimental results

Simulations

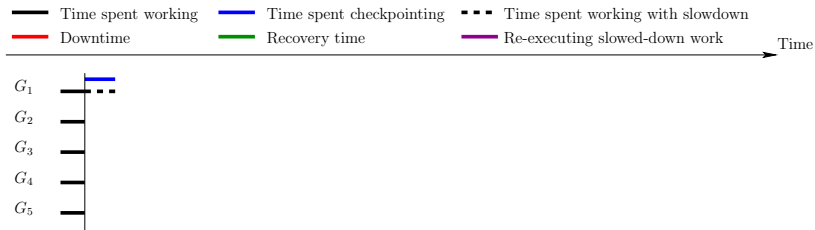
# Hierarchical checkpointing

- Processors partitioned into  $G$  groups
- Each group includes  $q$  processors
- Inside each group: coordinated checkpointing in time  $C(q)$
- Inter-group messages are logged

# Impact of checkpointing

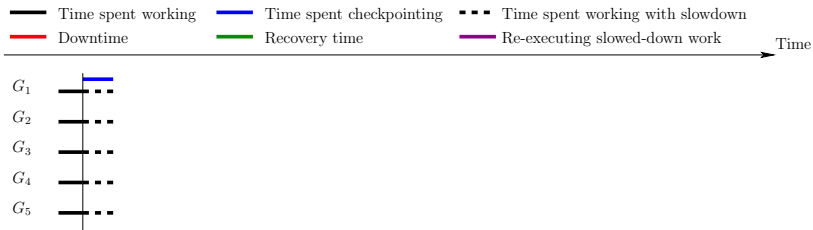


# Impact of checkpointing



When a group checkpoints, its own computation speed is slowed-down

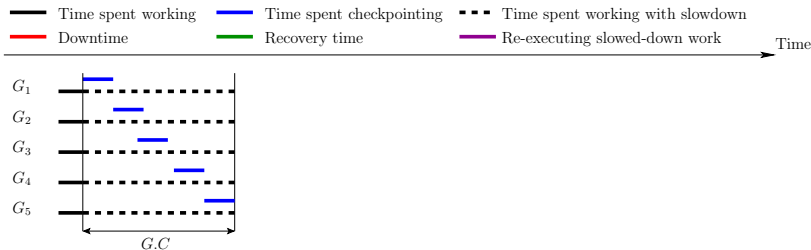
# Impact of checkpointing



When a group checkpoints, its own computation speed is slowed-down

This holds for all groups because of the tightly-coupled assumption

# Impact of checkpointing

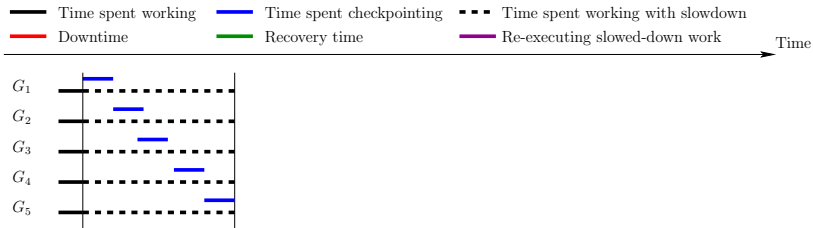


When a group checkpoints, its own computation speed is slowed-down

This holds for all groups because of the tightly-coupled assumption

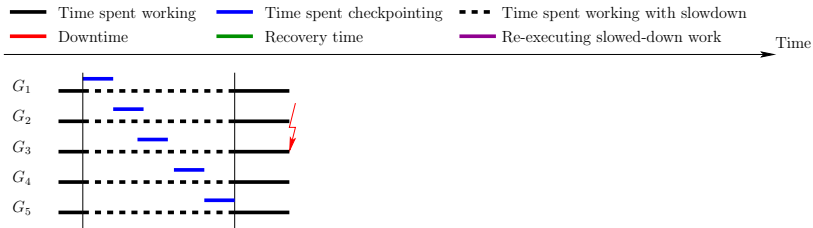
$$\text{WASTE}[FF] = \frac{T - \text{WORK}}{T} \text{ where } \text{WORK} = T - (1 - \alpha)GC(q)$$

# Failure during computation phase

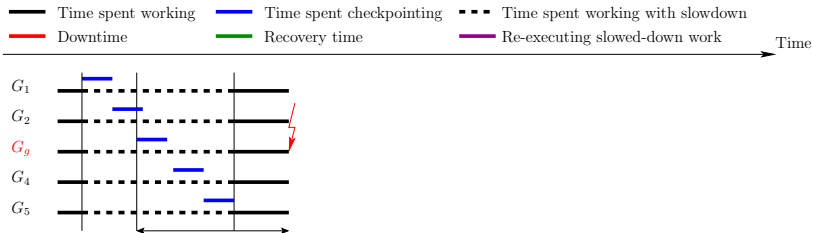




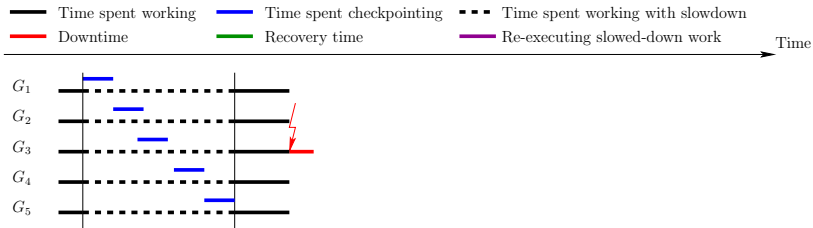
# Failure during computation phase



# Failure during computation phase

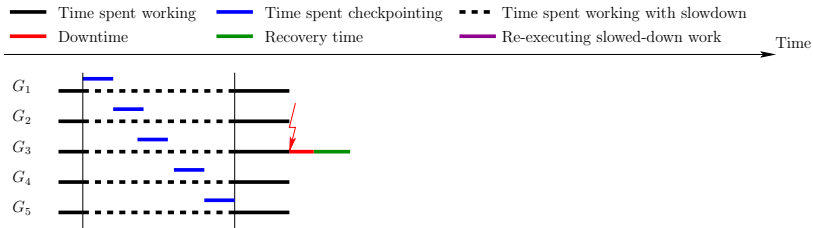


# Failure during computation phase



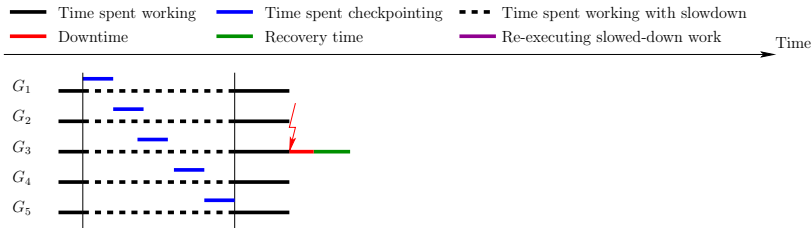
Tightly-coupled model: while one group is in downtime, none can work

# Failure during computation phase



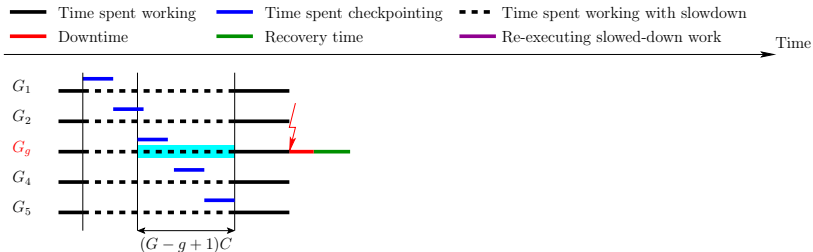
Tightly-coupled model: while one group is in recovery, none can work

# Failure during computation phase



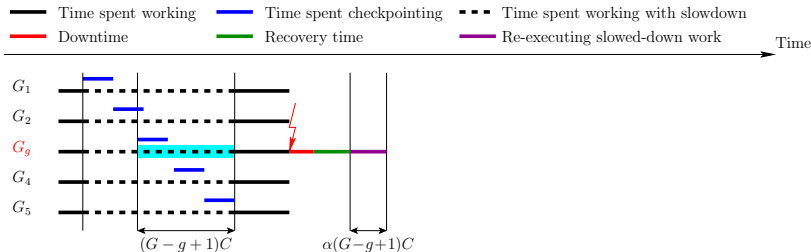
Groups must have completed the same amount of work in between two consecutive checkpoints, independently of the fact that a failure may have happened on the platform in between these checkpoints. Hence, no checkpointing is possible during the rollback.

# Failure during computation phase



Redo work done during previous checkpointing phase and that was destroyed by the failure

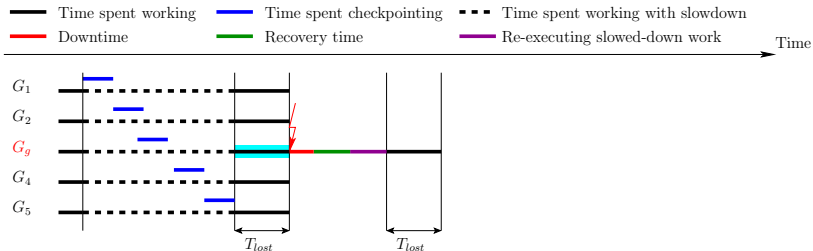
# Failure during computation phase



Redo work done during previous checkpointing phase and that was destroyed by the failure

But no checkpoint is taken in parallel, hence this re-computation is faster than the original computation

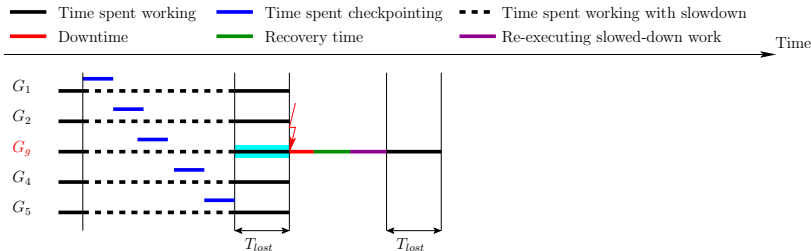
# Failure during computation phase



Redo work done in computation phase and that was destroyed by the failure



# Failure during computation phase

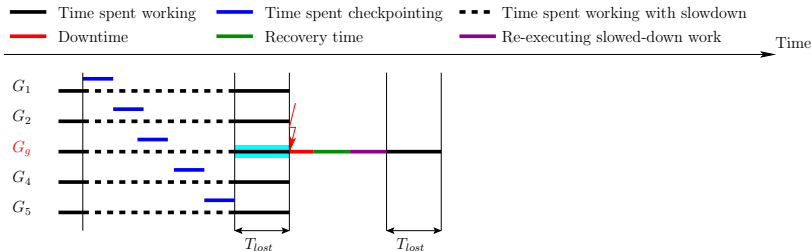


$$\text{RE-EXEC: } T_{lost} + \alpha(G - g + 1)C$$

$$\text{Expectation: } T_{lost} = \frac{1}{2}(T - G.C)$$

$$\text{Approximated RE-EXEC: } \frac{T - G.C}{2} + \alpha(G - g + 1)C$$

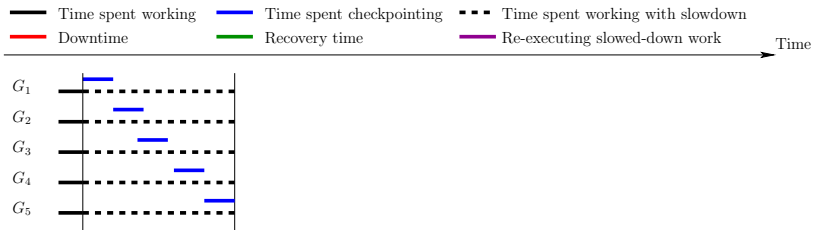
# Failure during computation phase



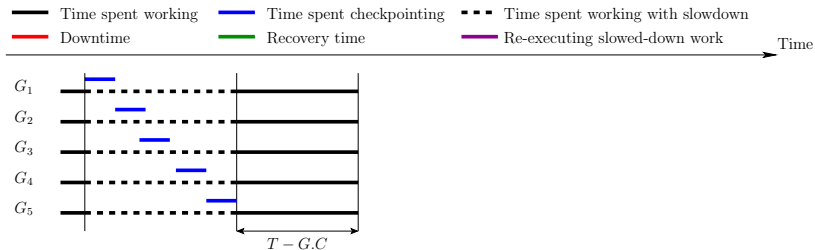
Average approximated RE-EXEC:

$$\begin{aligned}
 & \frac{1}{G} \sum_{g=1}^G \left[ \frac{T - G.C(q)}{2} + \alpha(G - g + 1)C(q) \right] \\
 &= \frac{T - G.C(q)}{2} + \alpha \frac{G + 1}{2} C(q)
 \end{aligned}$$

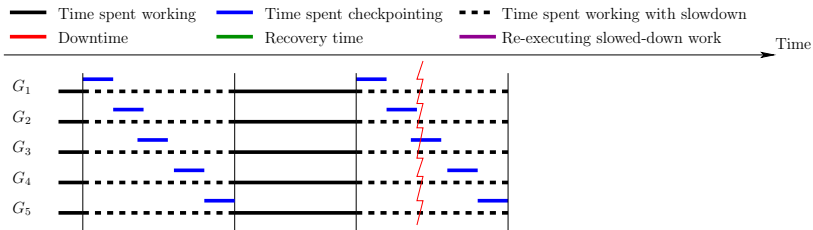
# Failure during checkpointing phase



# Failure during checkpointing phase



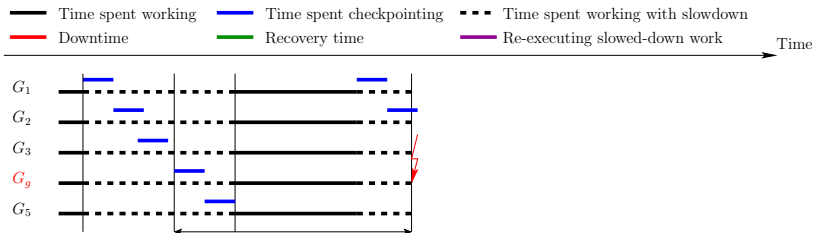
# Failure during checkpointing phase



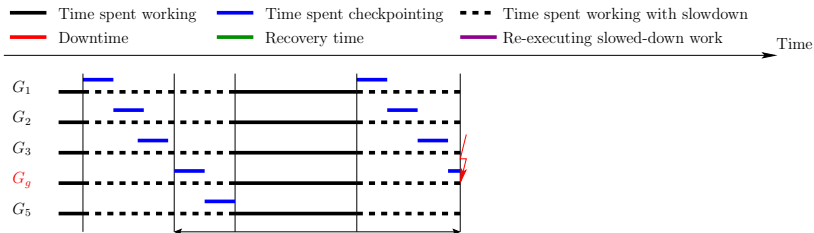
When does the failing group fail?

- ① Before starting its own checkpoint
- ② While taking its own checkpoint
- ③ After completing its own checkpoint

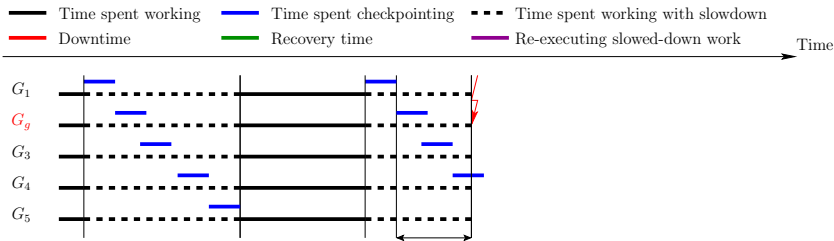
# Failure during checkpointing phase: failure before checkpoint



# Failure during checkpointing phase: failure during checkpoint



# Failure during checkpointing phase: failure after checkpoint





# Average waste for failures during checkpointing phase

Average RE-EXEC when the failing-group  $g$  fails

Overall average RE-EXEC:  $\text{RE-EXEC}_{ckpt} =$

$$\frac{1}{G}((g-1).\text{RE-EXEC}_{before\_ckpt} + 1.\text{RE-EXEC}_{during\_ckpt} + (G-g).\text{RE-EXEC}_{after\_ckpt})$$

Average over all groups:

$$\text{AVG\_RE-EXEC}_{ckpt} = \frac{G+1}{2G}T + \frac{\alpha C(q)(G+3)}{2} + \frac{C(q)(1-2\alpha)}{2G} - \frac{C(q)(G+1)}{2}$$

# Total waste

$$\text{WASTE}[FF] = \frac{T - \text{WORK}}{T} \text{ with } \text{WORK} = T - (1 - \alpha)GC(q)$$

$$\text{WASTE}[fail] = \frac{1}{\mu} \left( D(q) + R(q) + \text{RE-EXEC} \right) \text{ with}$$

$$\text{RE-EXEC} = \frac{T - GC(q)}{T} \text{RE-EXEC}_{comp} + \frac{GC(q)}{T} \text{RE-EXEC}_{ckpt}$$

$$\text{WASTE} = \text{WASTE}[FF] + \text{WASTE}[fail] - \text{WASTE}[FF]\text{WASTE}[fail]$$

Minimize WASTE subject to:

- $GC(q) \leq T$  (by construction)
- Gets complicated! Use computer algebra software ☹️

# Outline

- ① Protocol Overhead
- ② Accounting for message logging
- ③ Instantiating the model
- ④ Experimental results

# Impact on work

- ☹️ Logging messages slows down execution:  
 $\Rightarrow \text{WORK becomes } \lambda \text{WORK, where } 0 < \lambda < 1$   
 Typical value:  $\lambda \approx 0.98$
- 😊 Re-execution after a failure is faster:  
 $\Rightarrow \text{RE-EXEC becomes } \frac{\text{RE-EXEC}}{\rho}, \text{ where } \rho \in [1..2]$   
 Typical value:  $\rho \approx 1.5$

$$\text{WASTE}[FF] = \frac{T - \lambda \text{WORK}}{T}$$

$$\text{WASTE}[fail] = \frac{1}{\mu} \left( D(q) + R(q) + \frac{\text{RE-EXEC}}{\rho} \right)$$

# Impact on checkpoint size

- Inter-groups messages logged continuously
- Checkpoint size increases with amount of work executed before a checkpoint
- $C_0(q)$ : Checkpoint size of a group without message logging

$$C(q) = C_0(q)(1 + \beta \text{WORK}) \Leftrightarrow \beta = \frac{C(q) - C_0(q)}{C_0(q) \text{WORK}}$$

$$\text{WORK} = \lambda(T - (1 - \alpha)GC(q))$$

$$C(q) = \frac{C_0(q)(1 + \beta\lambda T)}{1 + GC_0(q)\beta\lambda(1 - \alpha)}$$

# Outline

- ① Protocol Overhead
- ② Accounting for message logging
- ③ Instantiating the model
- ④ Experimental results

# Three case studies

## Coord-IO

Coordinated approach:  $C = C_{\text{Mem}} = \frac{\text{Mem}}{b_{io}}$

where Mem is the memory footprint of the application

## Hierarch-IO

Several (large) groups, *I/O-saturated*

⇒ groups checkpoint sequentially

$$C_0(q) = \frac{C_{\text{Mem}}}{G} = \frac{\text{Mem}}{Gb_{io}}$$

## Hierarch-Port

Very large number of smaller groups, *port-saturated*

⇒ some groups checkpoint in parallel

Groups of  $q_{\min}$  processors, where  $q_{\min} b_{port} \geq b_{io}$

## ① Protocol Overhead

Coordinated checkpointing

Hierarchical checkpointing

## ② Accounting for message logging

## ③ Instantiating the model

Applications

Platforms

## ④ Experimental results

Simulations



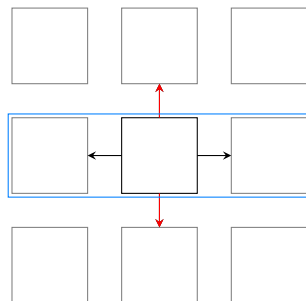
# Three applications

- ① 2D-stencil
- ② 3D-Stencil
  - Plane
  - Line
- ③ Matrix product

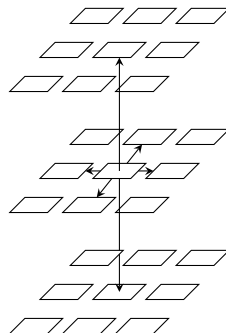
# Computing $\beta$ for Stencil-2D

$$C(q) = C_0(q) + \text{Logged\_Msg} = C_0(q)(1 + \beta \text{WORK})$$

- 2 out of the 4 messages are logged

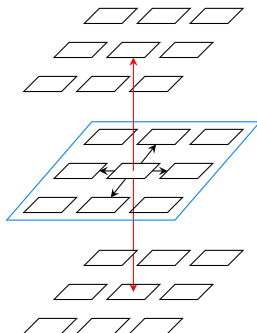


## Three applications: 2) 3D-stencil



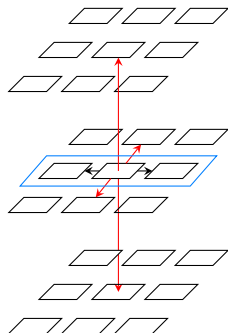
## Three applications: 2) 3D-stencil

- 3D-Plane: Vertical messages are logged



## Three applications: 2) 3D-stencil

- 3D-Plane: Vertical messages are logged
- 3D-Line: Twice as many messages are logged



## ① Protocol Overhead

Coordinated checkpointing

Hierarchical checkpointing

## ② Accounting for message logging

## ③ Instantiating the model

Applications

Platforms

## ④ Experimental results

Simulations

# Four platforms: basic characteristics

Name	Number of cores	Number of processors $p_{total}$	Number of cores per processor	Memory per processor	I/O Network Bandwidth ( $b_{io}$ )		I/O Bandwidth ( $b_{port}$ ) Read/Write per processor
					Read	Write	
Titan	299,008	16,688	16	32GB	300GB/s	300GB/s	20GB/s
K-Computer	705,024	88,128	8	16GB	150GB/s	96GB/s	20GB/s
Exascale-Slim	1,000,000,000	1,000,000	1,000	64GB	1TB/s	1TB/s	200GB/s
Exascale-Fat	1,000,000,000	100,000	10,000	640GB	1TB/s	1TB/s	400GB/s

# Four platforms: 2D-STENCIL and MATRIX-PRODUCT

Name	Scenario	$G(C(q))$	$\beta$ for 2D-STENCIL	$\beta$ for MATRIX-PRODUCT
Titan	COORD-IO	1 (2,048s)	/	/
	HIERARCH-IO	136 (15s)	0.0001098	0.0004280
	HIERARCH-PORT	1,246 (1.6s)	0.0002196	0.0008561
K-Computer	COORD-IO	1 (14,688s)	/	/
	HIERARCH-IO	296 (50s)	0.0002858	0.001113
	HIERARCH-PORT	17,626 (0.83s)	0.0005716	0.002227
Exascale-Slim	COORD-IO	1 (64,000s)	/	/
	HIERARCH-IO	1,000 (64s)	0.0002599	0.001013
	HIERARCH-PORT	200,000 (0.32s)	0.0005199	0.002026
Exascale-Fat	COORD-IO	1 (64,000s)	/	/
	HIERARCH-IO	316 (217s)	0.00008220	0.0003203
	HIERARCH-PORT	33,333 (1.92s)	0.00016440	0.0006407



# Four platforms: 2D-STENCIL and MATRIX-PRODUCT

Name	Scenario	$G(C(q))$	$\beta$ for 2D-STENCIL	$\beta$ for MATRIX-PRODUCT
Titan	COORD-IO	1 (2,048s)	/	/
	HIERARCH-IO	136 (15s)	0.0001098	0.0004280
	HIERARCH-PORT	1,246 (1.6s)	0.0002196	0.0008561
K-Computer	COORD-IO	1 (14,688s)	/	/
	HIERARCH-IO	296 (50s)	0.0002858	0.001113
	HIERARCH-PORT	17,626 (0.83s)	0.0005716	0.002227
Exascale-Slim	COORD-IO	1 (64,000s)	/	/
	HIERARCH-IO	1,000 (64s)	0.0002599	0.001013
	HIERARCH-PORT	200,000 (0.32s)	0.0005199	0.002026
Exascale-Fat	COORD-IO	1 (64,000s)	/	/
	HIERARCH-IO	316 (217s)	0.00008220	0.0003203
	HIERARCH-PORT	33,333 (1.92s)	0.00016440	0.0006407

# Four platforms: 3D-STENCIL

Name	Scenario	$G$	$\beta$ for 3D-STENCIL
Titan	COORD-IO	1	/
	HIERARCH-IO-PLANE	26	0.001476
	HIERARCH-IO-LINE	675	0.002952
	HIERARCH-PORT	1,246	0.004428
K-Computer	COORD-IO	1	/
	HIERARCH-IO-PLANE	44	0.003422
	HIERARCH-IO-LINE	1,936	0.006844
	HIERARCH-PORT	17,626	0.010266
Exascale-Slim	COORD-IO	1	/
	HIERARCH-IO-PLANE	100	0.003952
	HIERARCH-IO-LINE	10,000	0.007904
	HIERARCH-PORT	200,000	0.011856
Exascale-Fat	COORD-IO	1	/
	HIERARCH-IO-PLANE	46	0.001834
	HIERARCH-IO-LINE	2,116	0.003668
	HIERARCH-PORT	33,333	0.005502

# Outline

- ① Protocol Overhead
- ② Accounting for message logging
- ③ Instantiating the model
- ④ Experimental results

## ① Protocol Overhead

Coordinated checkpointing

Hierarchical checkpointing

## ② Accounting for message logging

## ③ Instantiating the model

Applications

Platforms

## ④ Experimental results

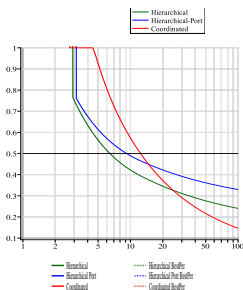
Simulations

# Simulation parameters

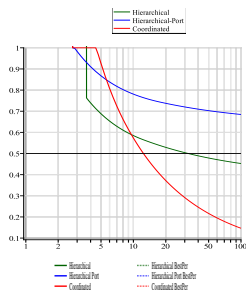
- Failure distribution: Weibull,  $k = 0.7$
- Failure free execution on each process: 4 days
- Time-out: 1 year
- No assumption on failures
- $\alpha = 0.3$ ,  $\rho = 1.5$ ,  $\lambda = 0.98$
- Each point: average over 20 randomly generated instances
- Computed period and best period:
  - Generate 480 periods in the neighborhood of the period from the model
  - Numerically evaluate the best one through simulations

# Platform: Titan

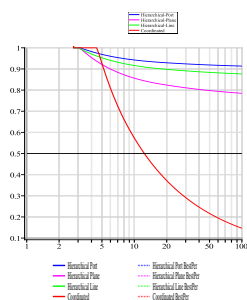
- Solid line: Computed period
- Dotted line: Best Period



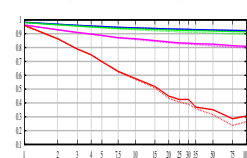
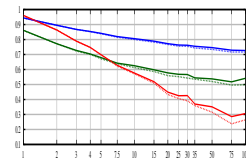
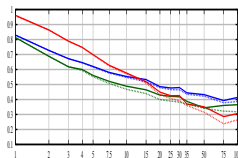
2D-Stencil



Matrix Product



3D-Stencil



Waste as a function of processor MTBF  $\mu$

# Platform: Exascale

WASTE = 1 for all scenarios!!!

## Platform: Exascale

WASTE = 1 for all scenarios!!!

Goodbye Exascale?!



# Checkpoint size for K-Computer and Exascale platforms

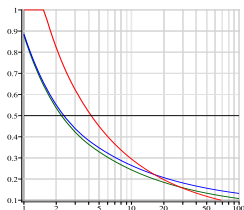
Name	$G$
K-Computer	14,688s
Exascale-Slim	64,000
Exascale-Fat	64,000

- Large time to dump the memory
- Using  $1\%C$ 
  - faster I/O and storage (two-level checkpoint, SSD, ...)
  - smaller amount of memory written
- Comparing with  $0.1\%C$  for the exascale platforms

# Platform: KComputer

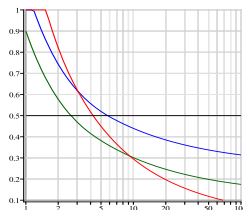
- Solid line: Computed period
- Dotted line: Best Period

— Hierarchical  
— Hierarchical-Port  
— Coordinated



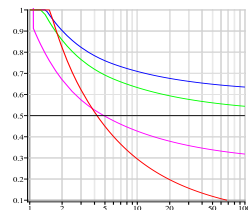
2D-Stencil

— Hierarchical  
— Hierarchical-Port  
— Coordinated



Matrix Product

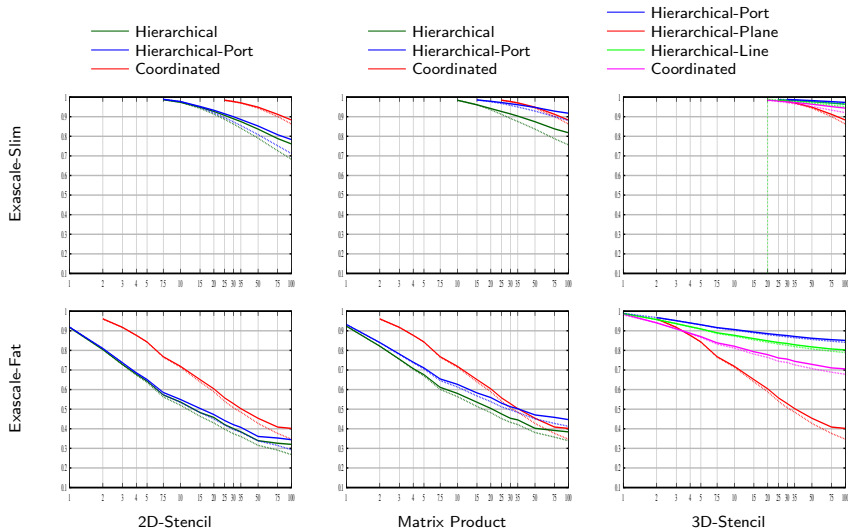
— Hierarchical-Port  
— Hierarchical-Plane  
— Hierarchical-Line  
— Coordinated



3D-Stencil

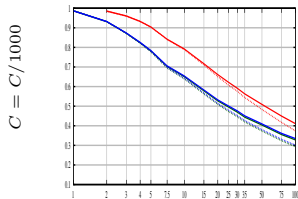
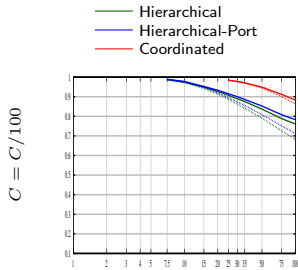
# Platform: Exascale with $C = C/100$

- Solid line: Computed period
- Dotted line: Best Period

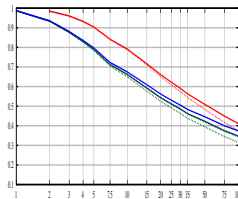
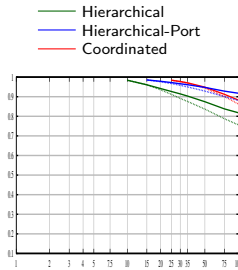


# Checkpoint impact: Exascale Slim

- Solid line: Computed period
- Dotted line: Best Period

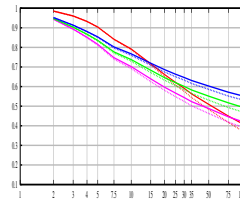
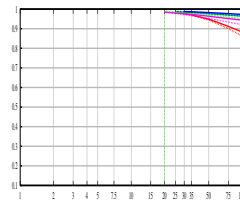


2D-Stencil



Matrix Product

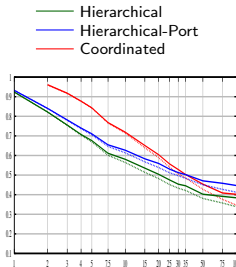
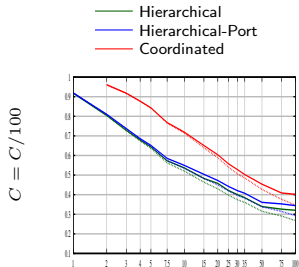
- Hierarchical-Port  
— Hierarchical-Plane  
— Hierarchical-Line  
— Coordinated



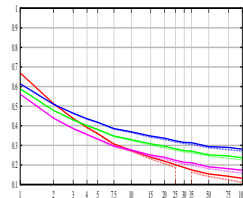
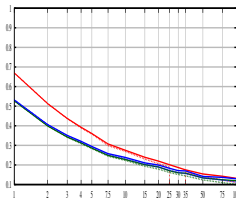
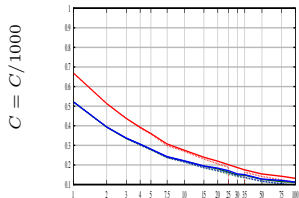
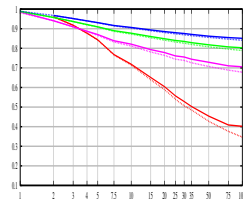
3D-Stencil

# Checkpoint impact: Exascale Fat

- Solid line: Computed period
- Dotted line: Best Period



- Hierarchical-Port
- Hierarchical-Plane
- Hierarchical-Line
- Coordinated



2D-Stencil

Matrix Product

3D-Stencil

Waste as a function of processor MTBF  $\mu$  with checkpoint variation

# Conclusion

- First attempt at analytical comparison of coordinated and hierarchical checkpointing protocols
- Classical models (Young, Daly) extended
  - Several new parameters ( $\alpha$ ,  $\lambda$ ,  $\rho$ )
  - Message logging impact ( $\beta$ )
- Instantiation
  - Scenarios: COORD-IO, HIERARCH-IO, HIERARCH-PORT
  - Realistic application/platform combinations
- Current work: Application co-scheduling
- Future work and possible collaboration:
  - Use trace-based failure logs
  - Application-dependant checkpointing

# Unified Model for Assessing Checkpointing Protocols at Extreme-Scale

George BOSILCA<sup>1</sup>, Aurélien BOUTEILLER<sup>1</sup>,  
Elisabeth BRUNET<sup>2</sup>, Franck CAPPELLO<sup>3</sup>,  
Jack DONGARRA<sup>1</sup>, Amina GUERMOUCHE<sup>4</sup>,  
Thomas HÉRAULT<sup>1</sup>, Yves ROBERT<sup>1,4</sup>,  
Frédéric VIVIEN<sup>4</sup>, and Dounia ZAIDOUNI<sup>4</sup>

1. University of Tennessee Knoxville, USA
2. Telecom SudParis, France
3. INRIA & University of Illinois at Urbana Champaign, USA
4. Ecole Normale Supérieure de Lyon & INRIA, France

November 21, 2012